Gravitational Field Due to a Sphere: A Geometrical Argument

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In a recent article[1], the author L. Ruby presented a method to calculate the gravitational force on a test mass due to a thin spherical shell. His point was not to use calculus. However, shifting from analytical to numerical integration does not much help the mathematically challenged student. In this comment, I would like to give a simple geometrical argument, that the field inside a thin spherical shell vanishes.

Argument for an Inside Point

Consider a point P_1 inside a thin spherical shell with homogeneous mass distribution (see Fig. 1). A double cone with center at P_1 cuts out two Areas A_1 and A_2. If the solid angle of the cone is infinitesimally small, the areas relate to the distances r_1 and r_2 as follows:

$$\frac{A_1}{A_2} = \frac{r_1^2}{r_2^2} \quad (1)$$

The gravitational fields g_1 and g_2 of these areas at point P_1 are:

$$g_1 \quad \frac{A_1}{r_1^2} = \frac{A_2}{r_2^2} \quad g_2 \quad (2)$$

The fields are equal in strength but have opposite directions. This argument applies regardless of the orientation of the cone. Therefore the field inside the spherical shell vanishes.

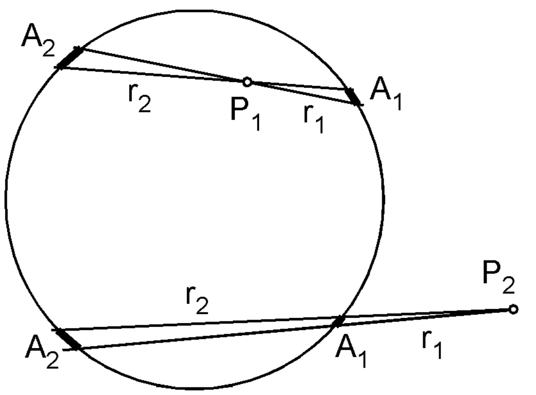

Fig. 1: Geometry for a point P_1 inside and a point P_2 outside a thin spherical shell.

Argument for an Outside Point

I did not find a way to by-pass an analytical or numerical integral to show that the gravitational field of a sphere is the same as that of a point mass in the center of the sphere. But we can give an argument against the opinion, that an area A_1 closer to the outside point P_2 (see Fig. 1) generates a stronger field at P_2 than an area A_2, which is farther away. Equations (1) and (2) hold as well for an outside point!

Reference

[1] L. Ruby, "Gravitational Force Due to a Sphere: A Non-calculus Calculation", Phys. Teach. **41**, 416-418 (Oct. 2003)